

E. M. Landis. **Three balls theorem for some nonlinear elliptic equations and its applications**
 // Nelinejnye granichnye zadachi (Nonlinear Boundary Value Problems). – 1989. – **1**. – p. 67-72.

The equation $Lu = u\varphi(|u|)$ is considered, where L is a linear uniformly elliptic operator of the second order. Conditions are imposed on the function φ , under which the three balls theorem is valid in the following statements: 1) the weak three balls theorem: there exists a constant $\sigma > 0$ such that if $0 < r < \frac{1}{2}$ and in the ball $|x| < 1$ the solution $u(x)$ is defined such that $|u(x)|_{|x|<1} < 1$ and $|u(x)|_{|x|<r} < \varepsilon$, then $|u(x)|_{|x|<2r} < \varepsilon^\sigma$ ($\varepsilon > 0$ is small enough). It is shown that the condition imposed on φ is close to the exact one; 2) the strong three balls theorem: there exist constants $C > 1, \alpha_0 > 1, k_0 > 1$ such that if $0 < r_1 < r < 1$ and $u(x)$ is the solution in the ball $|x| < 1$, $|u(x)|_{|x|<1} < 1, |u(x)|_{|x|<r_1} < r_1^\alpha$ for $\alpha > \alpha_0$, then $|u(x)|_{|x|<r} < (Cr)^{\alpha-k_0}$.

From these theorems a number of consequences on a possible rate of decay of nontrivial zero solution in unbounded domains of various shapes are deduced. A theorem similar to the weak three balls theorem for the case where the solution is growing rapidly at the transition from the internal ball to external one is given: there exists $\tau > 1$ such that if

$$|u(x)|_{|x|<r<\frac{1}{2}} < 1, \quad \sup_{|x|<2r} |u(x)| > M > M_0,$$

where M_0 is large enough, then

$$\sup_{|x|<1} |u(x)| < M^\tau.$$

By means of this theorem, a Phragmen- Lindelof type theorem is obtained.