P. E. Sobolevskii. **A theorem on mixed derivatives and an estimate of the rate of convergence of Euler-Galerkin method for parabolic equations** // Nelinejnye granichnye zadachi (Nonlinear Boundary Value Problems). – 1989. – **1.** – p. 97-102.

The Cauchy problem

$$v'(t) + Av(t) = f(t) \quad (0 \le t \le 1), \ v(0) = v^0, \tag{1}$$

is considered in a Hilbert space H. Here A is a positively defined self-adjoint (unbounded) operator. Let $\{e_k\}_{k=1}^{\infty}$ be a system of linearly independent elements of $\mathcal{D}\left(A^{\frac{1}{2}}\right)$, whose finite linear combinations are dense in $\mathcal{D}\,(A^{\frac{1}{2}})$. Let H^n be a linear span of elements $e_1, \dots, e_n$, P^n be an orthogonal projection in $\mathcal{D}\left(A^{\frac{1}{2}}\right)$ onto H^n. Differential problem (1) is associated with the projection-difference problem

$$(u_k^n - u_{k-1}^n)\,\tau^{-1} + A^n u_k^n = P^n f_k \quad (k = 1, \dots, N), \ u_0 = Q^n v^0, \ A^n\colon H^n \to H^n,$$

$$(A^n\varphi, \psi) = A^{\frac{1}{2}}\varphi, A^{\frac{1}{2}}\psi) \ \ \forall \varphi, \psi \in H^n; \ \tau = N^{-1}, f_k = \frac{1}{\tau}\int_{t_{k-1}}^{t_k} f(s)\,ds, t_k = k\tau. \tag{2}$$

The following estimate of the rate of convergence of method (2) is obtained:

$$\| \left\{A^{\frac{1}{2}}[u_k^n - v(t_k)]\right\}_{k=1}^{N} \|_{\alpha,N} \le$$

$$\le \left[\tau^{\frac{1}{2}} + \|(I - Q^n)A^{\frac{1}{2}}\|_{H\to H}\right] M\alpha^{-2}(1-\alpha)^{-2}\,[\|Av^0\|_H + \alpha^{-1}(1-\alpha)^{-1}\|f\|_\alpha]. \tag{3}$$

Here the norm $\|\cdot\|_\alpha$ is defined by the formula

$$\|f\|_\alpha = \max_{0\le t\le 1}\|f(t)\|_H + sup_{0<t<t+\Delta t\le 1}\|f(t+\Delta t) - f(t)\|_H \ (\Delta t)^{-\alpha}\, t^\alpha \ (0 < \alpha < 1), \tag{4}$$

and $\|\cdot\|_{\alpha,N}$ is induced by $\|\cdot\|_\alpha$ in the space H^n.