# ON GENERALIZED SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR SOME GENERAL DIFFERENTIAL EQUATIONS.

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In this article we will introduce and consider a notion of generalized solutions of boundary value problems for the equations of the following general form

$$\mathcal{L}^{+} \circ A \circ \mathcal{L} u = f \tag{1}$$

in arbitrary fixed domain $\Omega \subset \mathbb{R}^n$, where

$$\mathcal{L} = \sum_{|\alpha| \le m} a_\alpha(x) D^\alpha, D^\alpha = (-i\partial)^{|\alpha|}/\partial x_1^{\alpha_1} ... \partial x_n^{\alpha_n}, \alpha \in \mathbb{Z}_+^n, |\alpha| = \sum_k \alpha_k$$

is a differential operation with complex $j \times k$-matrix coefficients $a_\alpha(x)$, all elements of which are $C^\infty(\bar{\Omega})$-functions depending on $x \in \bar{\Omega}$, $\mathcal{L}^+ = \sum_{|\alpha| \le m} D^\alpha(a_\alpha^*(x)\cdot)$, $a_\alpha^* = \overline{a_\alpha^T}$ is the formally adjoint differential operation; and $A : L_2^k(\Omega) \longrightarrow L_2^k(\Omega)$ is some continuous (in general, nonlinear) operator of an arbitrary origin. The equations of such form were studing as a rule due to examinations of quasilinear differential equations on the whole of the elliptic type (see, for example, the books [1,2]). Note that every quasilinear differential equation $\sum\limits_{|\alpha| \le m} D^\alpha A_\alpha(x, u, \nabla u, ..., D^m u) = f$ can be considered as an equation (1), if the Nemytsky's operator $A : (u; u_1, ..., u_n;\ u_{11}, ..., u_{nn}; ... u_{nn...n}) \longrightarrow \{A_\alpha(x, u_1, ..., u_n; u_{11}, ..., u_{nn}; ... u_{nn...n})\}$ is an continuous operator from $L_2^N(\Omega)$ into itself, where $N$ be the number of all $\alpha, |\alpha| \le m$; in this case $\mathcal{L}u = (u, \nabla u, ..., D^m u)$. Note also that arguments by M.Vishik [3] and H.Gajevski [1] suggest a way of the study of boundary value problems for the equations (1) with an arbitrary operation $\mathcal{L}$.

On this way we shall need some definitions and facts from the general theory of boundary value problems.

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## 1. Some definitions of general theory.

We call to mind general facts about extensions of the differential operator and boundary value problems in the domain (see $[3,4,5,6]$).

The closing of the operator, which is given on the space $C_0^{\infty,j}(\Omega)$, consisting of finite smooth vector functions, by means of the operation $\mathcal{L}$, in the norm of the graph $\|u\|_L^2 = \|u\|^2_{L_2^j(\Omega)} + \|\mathcal{L}u\|^2_{L_2^k(\Omega)}$ is called the **minimal** expansion of the operator $\mathcal{L}$ in the space $L_2^j(\Omega)$ or simply the minimal operator $L_0$. The contraction of the operator, wich is generated by the operation $\mathcal{L}$ in the space $\mathcal{D}'(\Omega)$, to the domain of the definition $D(L) = \{u \in L_2^j(\Omega) | \mathcal{L}u \in L_2^k(\Omega)\}$, $L = \mathcal{L}|_{D(L)}$ is said to be the **maximal** expansion of the operator $\mathcal{L}|_{C_0^\infty(\overline{\Omega})}$ or simply the maximal operator $L$. Note that the space $D(L)$ is the Hilbert space with a scalar product of the norm $\| \ \|_L$ as well as his close subspace $D(L_0)$, which is the domain of the definition of the operator $L_0$. The kernel $\ker L$ is closed in the spaces $D(L)$ and $L_2^j(\Omega)$, the kernel $\ker L_0$ is closed in the spaces $D(L)$ and $\ker L$. Consider another expansion of the operator $\mathcal{L}|_{C^{\infty,j}(\overline{\Omega})}$, which we define $\tilde{L}$. This is an operator with a definition domain $D(\tilde{L})$, which is the closing of the space $C^{\infty,j}(\overline{\Omega})$ in the norm of the graph $\|\cdot\|_L$.

We consider the following conditions:

$$\text{the operator } L_0 : D(L_0) \to L_2^k(\Omega) \text{ has the continuous left-inverse;} \tag{2}$$

$$\text{the operator } L_0^+ : D(L_0^+) \to L_2^k(\Omega) \text{ has the continuous left-inverse;} \tag{3}$$

$$\tilde{L} = (L_0^+)^*; \tilde{L}^+ = (L_0)^*. \tag{4}$$

It is well known that $L = (L_0^+)^*$ and $L^+ = (L_0)^*$, so that the condition (4) means the equalities $D(L) = D(\tilde{L})$, $D(L^+) = D(\tilde{L}^+)$, i.e. the possibility to approximate each fuunction from $D(L)$ or $D(L^+)$ by functions from $C^{\infty,j}(\overline{\Omega})$. The conditions (2), (3) imply respectively the conditions: $\ker L_0 = 0, \ker L_0^+ = 0$. The conditions (2),(3),(4) was introduced in connection with the study of the concept of the correct posed boundary value problem which we remind here too (see [3,4,5,6]). We define the Cauchy space as $D(L)/D(L_0)$. In the paper [4] the Cauchy space was introduced as the factor $G(L)/G(L_0)$, where $G(L), G(L_0)$ are the graphs of the operators $L$ and $L_0$ res- pectively. It is not difficult to see that this definition is equivalent to entrodused. The homogeneous linear boundary value problem is by definition ([4]) the problem to find the solution $u \in D(L)$ of the relations

$$Lu = f, \ \Gamma u \in B, \tag{5}$$

where $\Gamma : D(L) \to C(L)$ is the map of the factorization, $B$ is a linear set in $C(L)$. The boundary value condition $\Gamma u \in B$ generates the subspace $D(L_B) = \Gamma^{-1}(B)$ of the space $D(L)$ and the operator $L_B$, which is a contraction of the operator $L$ on the spase $D(L_B)$ and which is an expantion of the operator $L_0$. This operator $L_B$ is closed if and only if the linear space $B$ is closed in $C(L)$ or the space $L_B$ is closed in $D(L)$ [4]. A boundary value problem is called correct posed and the operator $L_B$ is called solvable expansion of the operator $L_0$ if the operator $L_B : D(L_B) \to L_2^k(\Omega)$ has a two-sided inverse. The operator $L_1 : D(L_1) \to L_2^k(\Omega)$, which is the contraction of the operator $L$ (i.e. $D(L_1) \subseteq D(L)$), is called the solvable contraction if it has a two-sided inverse.

*Statement* 1. There exist a solvable expansion of the operator $L_0$ and there exist a correct posed boundary value problem for the equation $Lu = f$ if and only if the Vishik conditions (2) and (3) are fulfilled .

See the proof of this statement in the works of M.I.Vishik [3] and L.Hörmander [4]. Note that the same conditions (2) and (3) are equivalent to the exist of a correct posed boundary value problem for the equation $L^+u = f$.

We consider the following conditions too:

$$\text{the operator } L : D(L) \to L_2^k(\Omega) \text{ is surjective;} \tag{6}$$

$$\text{the operator } L^+ : D(L^+) \to L_2^j(\Omega) \text{ is surjective;} \tag{7}$$

$$\text{the operator } L : D(L) \to L_2^k(\Omega) \text{ is normally solvable.} \tag{8}$$

*Statement* 2 ([4]). The condition (2) is equivalent to the condition (7), the condition (3) is equivalent to the condition (6).

*Remark* 1. By virtue of this statement one could interpret the condition (7) as the fulfilment of the estimate $\|\varphi\|_{L_2(\Omega)} \leq C\|\mathcal{L}\varphi\|_{L_2(\Omega)}$ for all $\varphi \in C_0^\infty(\Omega)$, or, what is the same, $\|\varphi\|_{D(L)} \leq C_1\|\mathcal{L}\varphi\|_{L_2(\Omega)}$ with some $C_1 > 1$ , and it is analogously for (6). The condition (8) admits also such an interpretation. Namely, the fact that a linear continuous operator $M : H_1 \to H_2$ in Hilbert spaces is normally solvable, is equivalent to the fulfilment of the two-sided estimate $C_2\| u\|_{H_1} \leq\| Mu \|_{H_2}\leq C_3\| u\|_{H_1}$ on the orthogonal addition to $\ker M$, which is equivalent to the inequality $\exists C > 0, \forall u \in H_1, \| u\|^2_{H_1} - \| P_{\ker}u \|^2_{H_1}\leq C \| Mu \|^2_{H_2}$ (or the inequality $\exists C > 0, \forall u \in H_1, \| u\|_{H_1} - \| P_{\ker}u \|_{H_1}\leq C \| Mu \|_{H_2}$), where $P_{\ker} : H_1 \to \ker M$ is the orthogonal projector. Therefore, the condition (8) is equivalent to an estimate $\forall u \in D(L), \| u\|_{D(L)} - \|P_{\ker}u\|_{D(L)} \leq C\|Lu\|_{L_2(\Omega)}$ with some $C > 1$, or, what is the same, an inequality $\exists C > 0, \forall u \in D(L), \| u\|_{L_2(\Omega)} - \| P_{\ker} u\|_{L_2(\Omega)} \leq C \| Lu \|_{L_2(\Omega)}$, where one can suppose $u \in C^\infty(\overline{\Omega})$ if the condition (3) holds.

**2. Dirichlet problem.**

The function $u \in D(L_0)$, satisfying the integral identity

$$< A \circ L_0\, u,\, \mathcal{L}v >=< f, v > \tag{9}$$

for each function $v \in C_0^{\infty,j}(\Omega)$, will be called a generalized solution of the Dirichlet problem in the domain $\Omega$ for the equation (1) with $f \in D'(L)$ in the right-side part.

Note that the integral identity (9) is equivalent to the identity $< A\, L_0u,\, L_0v > = < f, v >, \forall v \in D(L_0)$, which means the following realisation of the equality (1):

$$L_0'\, A\, L_0\, u = f, \tag{10}$$

where $L_0' : L_2^k(\Omega) \to D'(L_0)$ is the dual operator to the operator $L_0 : D(L_0) \to L_2^k(\Omega)$.

If the domain $\Omega$ has the smooth boundary $\partial\Omega$ and the operator $A$ maps the space $C^{m,k}(\Omega)$ into itself then one can introduce a notion of the classical solution. The function $u \in C^{2m,j}(\Omega) \cap C^{m-1,j}(\overline{\Omega})$, satisfying the equation (1) with a function $f \in C^k(\Omega)$ and the boundary value conditions

$$u|_{\partial\Omega} = u'_\nu|_{\partial\Omega} = ... = u_\nu^{(m-1)}|_{\partial\Omega} = 0,$$

is called a classical solution of the Dirichlet problem in the domain $\Omega$ for the equation (1). It is obviously that the following statement is correct.

*Statement* 3. Each classical solution of the Dirichlet problem in the domain with the smooth boundary for the equation (1) with operator $A$ continuous maping from the space $C^{m,k}(\Omega)$ into itself is a generalized solution of this problem.

A generalized Dirichlet problem (9) shall be named correct posed or simply correct if the operator $L_0' \, A \, L_0 : D(L_0) \to D'(L_0)$ has the continuous two-sided inverse operator $M : D'(L_0) \to D(L_0)$. Let $P : L_2^k(\Omega) \to \mathrm{Im}\,(L_0)$ be the orthoprojector.

*Statement* 4. A generalized Dirichlet problem (9) is correct if and only if the condition (2) is fulfilled and the operator $P \circ A : \mathrm{Im}\,(L_0) \to \mathrm{Im}\,(L_0)$ is a homeomorphism.

This statement follows from more general statement 6, see below section 4.

*Example* 1. Consider the generalized Dirichlet problem for the Poisson equation $\Delta u = f : \mathcal{L} = grad, \mathcal{L}^+ = -div,\ D(L) = W_2^1(\Omega),\ D(L_0) = \overset{\circ}{W}{}_2^1(\Omega),\ f \in [\overset{\circ}{W}{}_2^1(\Omega)]'$. The statement 4 say, in particular, that such problem is correct posed in the arbitrary domain $\Omega$ if and only if in this domain one can prove the Fridrichs inequality: $\|\nabla u\| \geq C\|u\|, \forall u \in C_0^\infty(\Omega)$, which is in this case the Vishik condition (2) for the operator $\nabla$.

**3. Neumann problem.**

The function $u \in D(L)$, satisfying the integral identity

$$< A \circ Lu, \mathcal{L}v >=< f, v > \qquad (11)$$

for each function $v \in D(L)$, will be called a generalized solution of the Neumann problem in the domain $\Omega$ for the equation (1) with the arbitrary function $f \in D'(L)$.

If the condition (4) is fulfilled then it is sufficient to require the fulfillment of the integral identity (11) for each funtion $v \in C^{\infty,j}(\Omega)$. Note that the integral identity (11) is equivalent to the identity $< Lu, Lv >=< f, v >$, which means the realisation of the equality

$$L' \, A \, L \, u = f, \qquad (12)$$

where $L' : L_2^k(\Omega) \to D'(L)$ is the dual operator to the maximal operator $L : D(L) \to L_2^k(\Omega)$. This equality is, as a matter of fact, also a realization of the equation (1).

A generalized Neumann problem (11) shall be named normally correct if for each function $f \in D'(L)$, which is orthogonal to the space $\ker L$, there exists an unique to within an additive component $h \in \ker L$ the function $u \in D(L)$, which is a solution of the equation (12) and which continuous depend on $f$.

Let us denote by $P$ the orthoprojector from $L_2^k(\Omega)$ onto $\mathrm{Im}L$, which exists if the last subspace is closed.

*Statement* 5. A generalized Neumann problem (11) is normally correct if and only if the operator $L$ is normal solvable and the operator $P \circ A : \mathrm{Im}L \to \mathrm{Im}L$ is a homeomorphism.

This statement follows from more general statement, see below section 4. The operator $L$ is normally solvable, in particular, if it is fulfilled the condition (6).

Note that the Neumann problem in the domain with smooth boundary for the equation (1) with the smooth functions has the form: $A \circ Lu|_{\partial\Omega} = 0$. We shall need a notion of the conjugate to (5) boundary value problem, which is named the problem

$$L^+ v = g, \quad \Gamma^+ v \in B^+,$$

where $B^+ = \Gamma^+ D_B^+$, $D_B^+ = \{v \in D(L^+) | [u,v] = 0, \forall u \in \Gamma^{-1}(B)\}$. Here we use the Green's formula

$[u,v] := \int_\Omega (Lu \cdot \overline{v} - u \cdot \overline{L^+ v})\, dx = < \mathcal{L}_{\partial\Omega}\Gamma u,\, \Gamma^+ v >_{\partial\Omega} \; = \; - < \Gamma u, \mathcal{L}^+_{\partial\Omega}\Gamma^+ v >_{\partial\Omega}$. Note that in the last definition the domain $\Omega$ can be arbitrary.

*Example* 2. Consider the generalized Neumann problem for the Poisson equation $\Delta u = f$ (see the example 1). The statement 5 permits to state, in particular, that such problem is normally correct in the connected domain $\Omega$ with the finite-dimentional space of the first cogomologies $H^1(\Omega, \mathbb{R})$, for example, in the bounded domain with the smooth boundary (Here we have the same operators and spaces which are in the example 1 and the kernel and the cokernel of the operator of this problem are one-dimensional if the domain is connected). Indeed, by the de Rham theorem the closed in $L_2^n(\Omega)$ kernel of the operator *rot* (= the exterior differential $d_2$) include the image of the operator *grad* (= the exterior differential $d_1$) and the difference is a finite-dimensional space, therefore the space of the potential vector fields $\nabla H^1(\Omega)$ is closed.

On the other hand, as we show in the remark 1, the normal solvability of the operator $L$ is equivalent to the fulfilment of the inequality $\exists C > 0, \forall u \in D(L)$, $\| u\|^2_{L_2(\Omega)} - \|P_{\ker} u\|^2_{L_2(\Omega)} \le C\|Lu\|^2_{L_2(\Omega)}$,where $P_{\ker} : L_2(\Omega) \to \ker M$ is the orthogonal projector. For $L = \nabla$ we have $\ker L = \{const\}, P_{\ker} \; : \; u \to \frac{1}{\text{meas }\Omega}\int_\Omega u(x)\,dx$ and the last inequality in this case has the form of the well-known Poincare inequality: $\exists C > 0, \forall u \in C^\infty(\overline{\Omega})$, $\| u\|^2_{L_2(\Omega)} \le \frac{1}{\text{meas }\Omega}(\int_\Omega u\,dx)^2 + C\,\|\nabla u\|^2_{L_2(\Omega)}$. Thus, the statement 5 asserts that the generelized Neumann problem for Poisson equation is normally correct in a bounded domain $\Omega$ if and only if in this domain the Poincare inequality is fulfilled.

**4. Other boundary value problems.**

Let us consider the generalized setting of other boundary value problems. Assume that the space $B \subset C(L)$ give boundary value problem (5) and hence it gives an expansion $L_B$ of the minimal operator $L_0$. The function $u \in D(L)$, satisfying the integral identity

$$< A \circ L_B\, u,\, L_B\, v >=< f, v > \tag{13}$$

for each function $v \in D(L_B)$, will be called a generalized solution of the problem $\Gamma u \in B$, $\Gamma^+ ALu \in B^+$ ($B^+$ gives the conjugate problem, see s.3), generated of the problem (5), in the domain $\Omega$ for the equation (1) with the arbitrary function $f \in D'(L_B)$.

Note besides that the integral identity (13) means the validity of the equality

$$L'_B\, A\, L_B\, u = f, \tag{14}$$

where $L'_B : L_2^k(\Omega) \to D'(L_B)$ is the dual operator to the operator $L_B : D(L_B) \to L_2^k(\Omega)$, which define the equation (1) more exactly. Note also that by virtue of the density of the embedding $D(L_B) \subset L_2^j(\Omega)$ the space $L_2^j(\Omega)$ is dense embedded in the space $D'(L_B)$, therefore the solvability of the problem (13) with each function $f \in D'(L_B)$ imply its solvability with $f \in L_2^j(\Omega)$.

We shall call as usually an expansion $L_1$ of the minimal operator in the space $L_2(\Omega)$ (and the constraction of the maximal) normally solvable if the space $\text{Im}L_1$ is closed in $L_2(\Omega)$. We shall call the problem (13) normally correct if for each function $f \in D'(L_B)$, which is orthogonal to the space $\ker L_B$, there exists the unique to within an additive component $h \in \ker L_B$ function $u \in D(L_B)$, which is a solution of the equation (12), and which continuous depend of $f$. The generalized boundary value problem (13) for the equation (1) will be called correct if the operator $L'_B \circ A \circ L_B : D(L_B) \to D'(L_B)$

has the continuous two-sided inverse $M : D'(L_B) \to D(L_B)$. This definitions imply the following statement. Let us denote by $P$ the orthoprojector from $L_2^k(\Omega)$ onto $\mathrm{Im}L$, which exists if the last subspace is closed.

*Statement* 6. The generalized problem (13) for the equation (1) in the domain $\Omega$ is normally correct if and only if the operator $L_B$ is normally solvable, and the operator $P \circ A : \mathrm{Im}L \to \mathrm{Im}L$ is a homeomorphism. The generalized problem (13) for the equation (1) in the domain $\Omega$ is correct if and only if the operator $L_B$ is normally solvable and $\ker L_B = \{0\}$, and the operator $P \circ A : \mathrm{Im}L \to \mathrm{Im}L$ is a homeomorphism.

*Proof*. The sufficiency is obvious. Let us prove the necessity. Let the operator $L'_B\, A\, L_B$ be a homeomorphism, $M : D'(L) \to D(L)$ be its inverse and let the expansion $L_B$ be not normally solvable, i.e. the image of $L_B$ is not closed in $L_2^k(\Omega)$. Then there exists such element $g \in \mathrm{Im}' L_B$, which is not belong in the space $L_2(\Omega)$. On the other hand the element $A \circ L_B \circ M \circ L'_B\, g$ coincide with $g$, because this operator also is a homeomorphism. Thus, the operator $L_B$ is normally solvable. Futher, the operators $L'_B : \mathrm{Im}L_B \to D'(L_B)$ and $L_B : D(L_B) \to \mathrm{Im}L_B$ are homeomorphisms, and if we shall consider the operator $L_B$ as asking in the spaces $D(L_B) \to L_2^k(\Omega)$ then $L'_B : L_2^k(\Omega) \to D'(L_B)$ and $L'_B \circ P = L'_B$, therefore the operator $PA$ is a homeomorphism in the space $\mathrm{Im}L_B$.

The proof of the second part of this statement is analogous to the first part with such variation that the space $D(L)$ require the factorization by the space $\ker L$.

The statement 6 implies the following statement.

*Statement* 7. Assume that the expansion $L_B$ is solvable. Then the problem (13) is correct if and only if the operator $A : L_2^k(\Omega) \to L_2^k(\Omega)$ is a homeomorphism.

*Example* 3. Consider the generalized boundary value problem (13) for quasilinear equation (1) with scalar operators and functions, where $A : L_2(\Omega) \to L_2(\Omega)$ is some continuous mapping, for example, the Nemytsky's operator given by means of a map in $\mathbb{R}^1 : A(v)(x) = a(v(x))$. It is easily to see that

1) the homeomorphism of the map $a(x)$ in $\mathbb{R}^1$ and

2) linear growth on the infty: $(C_1|\xi| + C_2 \le a(\xi) \le C_3|\xi| + C_4)$

ensures the homeomorphism of the mapping $A$. The statement 7 gives the correctness of this problem if the problem (5) is correct. In particular, the following generalized boundary value problem is correct:

$$\Box a(\Box u) = f \in D'(\Box_B) \ \text{ in the unit disk } K,$$

$$u \in D(\Box_B), a(\Box u) \in D(\Box_{B^+}),\ \Box = \frac{\partial^2}{\partial x_1 \partial x_2}$$

where $D(\Box_B) = \{u \in D(\Box)|\ u(\tau) = 0 \ \text{ on } \Gamma_1 = \{\angle\tau \in \partial K, \frac{\pi}{2} \le \tau \le 2\pi\},\ u'_\nu(\tau) = 0 \ \text{ on } \Gamma_2 = \{\angle\tau \in \partial K, \pi \le \tau \le \frac{3\pi}{2}\}\},\ u(\tau) \in B; D(\Box_B^+) = \{u \in D(\Box)|\Gamma^+ u \in B^+\}$, $B^+ = \{u(\tau)|\ u(\tau) = 0 \ \text{ on } \complement\Gamma_1 = \{\angle\tau \in \partial K,\ 0 \le \tau \le \frac{\pi}{2}\},\ u'_\nu(\tau) = 0 \ \text{ on } \complement\Gamma_2 = \{\angle\tau \in \partial K, \frac{-\pi}{2} \le \tau \le \pi\}\}$ and the function $a$ has the properties 1), 2). The correctness of boundary value problem (5) with such $L_B = \Box_B$ proved in the work [7]. Note that the boundary value conditions $u|_{\Gamma_1} = 0, u'_\nu|_{\Gamma_2} = 0$ are understanded in the sense of the articles [8,9,10], i.e. as the vanishing of $L$-traces respectively of the zero and first order, which exist for any functions from the definition domain of the maximal operator $L = \Box$.

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