On the problem of $\mathbf{B}_0$-reduction for Navier-Stokes-Maxwell equations

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A question of investigation of the range of significances of parameters, at which to description of the flow of conducting fluid is possible to apply $\mathbf{B}_0$-reduction of Navier-Stokes-Maxwell equations (inductionless approximation) constantly involved attention of specialists in magnetic hydrodynamics. It was considered that application of $\mathbf{B}_0$-reduction is possible, when parameter R_m is reasonably small. A.B.Tsinober has paid attention that the intensity of magnetic field decrease with growth of Hartmann parameter. It has allowed to formulate hypothesis about of simultaneous admissibility of $\mathbf{B}_0$-reduction and Stokes approximation. On the necessity of the mathematical analysis of admissibility of $\mathbf{B}_0$-reduction author's attention has paid H.E.Kalis.

In this paper a range of applicability of $\mathbf{B}_0$-reduction for a rather wide class of MHD-flows in bounded plane domains is investigated. Besides effective formulas of estimates of absolute error are removed. Everywhere in this paper it should Hartmann and Reynolds numbers H, R are greater that 1.

Let $D = D^+ \cup D^-$ be a bounded plane domain. Boundary $S = \bigcup\limits_{i=0}^{n} S_i$ be a join of contours $S_i \in C_1^\alpha, \alpha > 0$. It separates domains D^+ and D^-. Domain D^- is bounded by $S^e = \bigcup\limits_{j=0}^{m} S_j^e$ and contours $S_j^e \in C_1^\alpha$. In $D \cup S$ Cartesian coordinates $x = (x_1, x_2)$ are entered. Let $D = D^+ \cup D^-$ be a bounded plane domain. Boundary $S = \bigcup\limits_{i=0}^{n} S_i$ be a join of contours $S_i \in C_1^\alpha, \alpha > 0$. It separates domains D^+ and D^-. Domain D^- is bounded by $S^e = \bigcup\limits_{j=0}^{m} S_j^e$ and contours $S_j^e \in C_1^\alpha$.

Let consider the Navier-Stokes-Maxwell equations in the dimensionless and coordinateless form:

in the domain D^+:

$$\begin{gathered}(\nabla\times)^2\,\mathbf{U} + \nabla\left(P + \tfrac{R}{2}|\mathbf{U}|^{\mathbf{2}}\right) = R\mathbf{U}\times(\nabla\times\mathbf{U})- \\ -H^2R_m^{-1}\mathbf{B}\times(\nabla\times\mathbf{B}); \\ \nabla\times\mathbf{B} = R_m\left(\mathbf{E}+\mathbf{U}\times\mathbf{B}\right); \\ \nabla\cdot\mathbf{U}=0, \quad \nabla\cdot\mathbf{B}=0; \quad \nabla\times\mathbf{E}=\mathbf{0};\end{gathered} \tag{1}$$

in the domain D^-:

$$\nabla\times\mathbf{B} = \mathbf{I}_0(x), \quad \nabla\cdot\mathbf{B}=0, \quad \nabla\times\mathbf{E}=\mathbf{0}. \tag{2}$$

Vector field $\mathbf{I}_0(x)$ is assumed known. Vector fields $\mathbf{U}, \mathbf{B}, \mathbf{E}$ satisfy to boundary value conditions:

$$\mathbf{U}_{|S} = \mathbf{U}_0, \quad \mathbf{U}_0\cdot\nu = 0; \quad \mathbf{B}\times\nu_{|S_0^e} = 0; \quad \mathbf{E}\times\nu_{|S} = \mathbf{0}; \tag{3}$$

where ν be a unit normal to S vector field. Let τ be a unit tangential to S vector field and $\kappa = \nu\times\tau$. Vector field $\mathbf{B}$ is assumed continuous on the S. In this conditions $\mathbf{E}\equiv\mathbf{0}$.

Let $\mathbf{B}_0$ be a solution of the problem (1)-(3) at $R_m = 0$. Later on will be assumed that vector field $\mathbf{B}_0$ satisfies to conditions:

$$0 < \underline{m} \le |\mathbf{B}_0| \le \overline{m} < \infty; \quad \int\limits_{S_0} |\mathbf{B}_0 \cdot \nu| \, dS_0 > 0. \tag{B}$$

Let $\beta_1 = |\mathbf{B}_0|^{-1} \mathbf{B}_0$; $\beta_2 \perp \beta_1, |\beta_2| = 1$, $\mu = \underline{m}\overline{m}^{-1} < 1$. Vector fields $(\beta_1, \ \beta_2)$ form in $D \cup S$ a local basis connected with vector field $\mathbf{B}_0$. Later on next designation will be used $\mu = \underline{m}^{-1}\overline{m} \le 1$.

By nonlinear inductionless approximation of the solution of the problem (1)-(3) is named a triple $(\mathbf{U}_l, P_l, \mathbf{E}_l)$, which satisfy in the domain D^+ to reduced Navier-Stokes-Maxwell system:

$$\begin{gathered}(\nabla\times)^2\, \mathbf{U}_l + \nabla\left(P_l + \tfrac{R}{2}|\mathbf{U}_l|^2\right) = R\mathbf{U}_l \times (\nabla \times \mathbf{U}_l) - \\ -H^2\mathbf{B}_0 \times (\mathbf{E}_l + \mathbf{U}_l \times \mathbf{B}_0); \\ \nabla \cdot \mathbf{U}_l = 0; \quad \nabla \times \mathbf{E}_l = 0; \end{gathered} \tag{4}$$

and boundary value conditions (3).

By the error of $\mathbf{B}_0$-reduction is named a triple $(\delta\mathbf{U} = \mathbf{U} - \mathbf{U}_l, \delta P = P - P_l, \delta\mathbf{B} = \mathbf{B} - \mathbf{B}_0)$. This triple satisfies to next system

$$\begin{gathered}(\nabla\times)^2\, \delta\mathbf{U} + \nabla\left[\delta P + \tfrac{R}{2}\left(2\mathbf{U}_l + |\delta\mathbf{U}|^2\right)\right] = R\left\{\delta\mathbf{U} \times (\nabla \times \mathbf{U}_l) + \right. \\ \left. + \mathbf{U}_l \times (\nabla\times\delta\mathbf{U}) + \delta\mathbf{U} \times (\nabla\times\delta\mathbf{U})\right\} - \\ -H^2\left[\mathbf{B}_0 \times (\mathbf{U}_l \times \delta\mathbf{B} + \delta\mathbf{U} \times \mathbf{B}_0 + \delta\mathbf{U}\times\delta\mathbf{B}) + \right. \\ \left. + \delta\mathbf{B} \times (\mathbf{U}_l \times \mathbf{B}_0 + \mathbf{U}_l \times \delta\mathbf{B} + \delta\mathbf{U} \times \mathbf{B}_0 + \delta\mathbf{U}\times\delta\mathbf{B})\right]; \\ \nabla\times\delta\mathbf{B} = R_m\left(\mathbf{U}_l \times \mathbf{B}_0 + \mathbf{U}_l \times \delta\mathbf{B} + \delta\mathbf{U} \times \mathbf{B}_0 + \delta\mathbf{U} \times \delta\mathbf{B}\right); \\ \nabla\cdot\delta\mathbf{U} = 0, \quad \nabla\cdot\delta\mathbf{B} = 0. \end{gathered} \tag{5}$$

and homogeneous boundary value conditions.

1. Some auxiliary results. Let $\mathbf{L}_p(D)$ - be a Banach space of vector fields $\mathbf{a}$, which have p-degree integrable module in D. If $p = 2$ this is a Hilbert's space and index $_p$ in that case will be missed. The scalar product in $\mathbf{L}_2$ is designated as $<,>$. Let $\mathbf{H}_\nu(D)$ - be a Hilbert space of solenoidal in D vector fields, which normal components are equal to a zero on S^- and have a finite norm:

$$\|\mathbf{a}\|_{H_\nu} = \|\nabla \times \mathbf{a}\| \,.$$

Theorem 1 *If $\mathbf{f} \in \mathbf{H}_\nu(D)$ and $q < 2n/(n-2), n$ be a dimension of D then next inequality is valid*

$$\|\mathbf{f}\|_q \le M_q \,\|\nabla \times \mathbf{f}\| \,.$$

A subspace of $\mathbf{H}_\nu(D)$, consisting of vector fields which are equal to a zero on S is denoted as $\mathbf{H}(D^+)$.

It is well known an outstanding role of the embedding theorems and it's expressions as multiplicative inequalities [1] in the theory of Navier- Stokes equations. Here some vector analog of this inequalities will be formulated without proof. Let (ξ_1, ξ_2) be some nondegenerate system of curvilinear coordinates in $D^+ \cup S$. Let Lame coefficients of this coordinate system are $\Lambda_1 = \Lambda_2 = \Lambda \ \left(0 \le \underline{\lambda} \le \Lambda \le \overline{\lambda}\right)$ and d be a diameter of D^+.

Theorem 2 *Let $q > 2$ and $\mathbf{f}(f_1, f_2) \in \mathbf{H}(D^+)$. Next inequalities are valid*

$$\begin{gathered} \|f_1\| \le d^{1/2} \|f_2\|^{1/2} \|\nabla \times \mathbf{f}\|^{1/2}, \\ \|f_1\|_r \le 4\overline{\lambda}\underline{\lambda}^{-1} \|f_1\|^{1-3\alpha} \|f_2\|^{\alpha} \|\nabla \times \mathbf{f}\|^{2\alpha}, \quad 2 < r < 6; \\ \|f_1\|_r \le \beta\overline{\lambda}\underline{\lambda}^{-1} \|f_2\|^{\alpha} \|\nabla \times \mathbf{f}\|^{1-\alpha}, \quad 6 < r < \infty; \\ \alpha = 1/2 - 1/r, \quad \beta = \max(4, r/2). \end{gathered} \tag{6}$$

Theorem 1 is given without proof.

Later on one special extension of vector field $\mathbf{U}_0$ into domain D^+ will be need. Let $\mathbf{a}_\delta$ be a twice differentiable in D^+ vector field, which is equal to $\mathbf{U}_0$ on S and to a zero in the points of D^+, distended from S greater then on δ. This vector field can be constructed by means of well known Hopf's cutoff function or it generalizations. Later on will be used one form of $\mathbf{a}_\delta$, constructed in [2]. Estimates for $\|\mathbf{a}_\delta\|_p, \|\nabla \times \mathbf{a}_\delta\|$, $p > 2$ contain parameter δ. For the purposes of this paragraph it is possible to choose $\delta = \delta_0 H^{-1}, \delta_0 = \min(1/3, K^{-1})$, K - be a maximum of curvatures of contours S_k. In view of this, estimates for $\|\mathbf{a}_\delta\|_p, \|\nabla \times \mathbf{a}_\delta\|$ will accept next form:

$$\begin{aligned} \|\mathbf{a}_\delta\|_p &\le 2^{1-2/p} M_\nabla \left[1 + 9(1+p)^{1/p}\right] L_s{}^{1/p} H^{-1/p} = C_{ap} H^{-1/p}; \\ \|\mathbf{a}_\delta\| &\le 18 M_\nabla L_s^{1/2} H^{-1/2} = C_a H^{-1/2}; \\ \|\nabla \times \mathbf{a}_\delta(x)\| &\le (26 M_\nabla + M_\delta)(L_s/\delta_0)^{1/2} H^{1/2} = C_{a\nabla} H^{1/2}, \end{aligned} \tag{7}$$

where M_∇, M_δ - some positive constants, L_s be a length of S.

For solutions of the boundary value problems (1)-(3) and it $\mathbf{B}_0$ -reduction next a priory estimates are valid

$$\begin{gathered} \|\nabla \times \mathbf{U}\| \le C_\nabla H^{1/2}, \qquad \|\mathbf{U} \times \mathbf{B}\| \le C_\perp H^{-1/2}, \\ \|\nabla \times \mathbf{B}\| = \|\nabla \times \delta\mathbf{B}\| \le R_m C_b H^{-1/2}; \\ \|\nabla \times \mathbf{U}_l\| \le C_{l\nabla} H^{1/2}, \qquad \|\mathbf{U}_l \times \beta_2\| \le C_{l\perp} H^{-1/2}, \\ \|\mathbf{U}_l\|_p \le C_{lp} H^{1/4-1/2p}. \end{gathered} \tag{8}$$

Lemma 1 *Let $S \in C_1^\alpha, 0 < \alpha < 1$, vector field $\mathbf{B}_0$ satisfies in $D^+ \cup S$ to condition (B) and $H \ge H_{01}$, where*

$$\begin{gathered} H_{01} > \left[\frac{M_c C_c R_m L_{S_0}^{-1}}{max\left(\overline{m} L_{S_0}, \int\limits_{S_0} |\mathbf{B}_0 \cdot \nu|^2 dS_0\right)} \right]^{11/5}, \\ C_c = \overline{m}\,(C_{ap} + C_b) + 4 M_r C_2 C_q R_m, \\ p > 2, \quad 1/r + 1/q = 1 \end{gathered}$$

and $L_{S_0}-$ be a length of contour S_0, M_c be a constant of embedding of Sobolev's space $W_p^1(D^+)$ into $C(D^+)$. Then vector field $\mathbf{B}_s$ satisfies in $D^+ \cup S$ to condition (B) with some constants $\underline{M}$, $\overline{M}$: $\quad \underline{M} \le |\mathbf{B}| \le \overline{M}$.

In conditions of Lemma 1 next a priory estimate for solutions of the problem (1)-(3) is valid

$$\|\mathbf{U}\|_p \le C_p H^{1/4-1/2p}.$$

A problem of finding of estimates of the error and conditions of admissibility of $\mathbf{B}_0$-reduction is reduced to obtaining of a priori estimates of solutions of the system (5) with homogenious boundary value conditions. As far as for $\|\nabla \times \delta\mathbf{B}\|$ estimate (8) is valid, reasonably will establish estimates of norms: $\|\nabla \times \delta\mathbf{U}\|$, $\|\delta\mathbf{U} \times \mathbf{B}_0\|$, $\|\delta\mathbf{U}\|$.

For obtaining of a priori estimates of solutions of the problem (5) by the standard way an equation of the balance of energy is made out:

$$\|\nabla\times\delta\mathbf{U}\|^2+H^2\|\delta\mathbf{U}\times\mathbf{B}_0\|^2=R\left\langle\nabla\times\delta\mathbf{U},\delta\mathbf{U}\times\mathbf{U}_l\right\rangle-\\ -H^2\left[\left\langle\delta\mathbf{U}\times\mathbf{B}_0,\mathbf{U}\times\delta\mathbf{B}\right\rangle+\left\langle\delta\mathbf{U}\times\delta\mathbf{B},\mathbf{U}\times\mathbf{B}\right\rangle\right].$$

In this equation vector fields $\mathbf{U}_l,\delta\mathbf{U},\delta\mathbf{B}$ are decomposed by vectors of the basis (β_1,β_2):

$$\mathbf{U}_l=U_{l1}\beta_1+U_{l2}\beta_2,\qquad \delta\mathbf{U}=w_1\beta_1+w_2\beta_2,\qquad \delta\mathbf{B}=\delta B_1\beta_1+\delta B_2\beta_2.$$

Summands, worth in both parts of equation of the balance of energy, are estimated by means of Hölder's inequality:

$$\|\nabla\times\delta\mathbf{U}\|^2+H^2\underline{m}^2\|w_2\kappa\|^2\ \le\ R\|\nabla\times\delta\mathbf{U}\|\left(\|w_1U_{l2}\kappa\|+\|w_2U_{l1}\kappa\|\right)+\\ +H^2\left(\overline{m}\,\|w_2\kappa\|\,\|\mathbf{U}\times\delta\mathbf{B}\|+C_\perp H^{-1/2}\,\|\delta\mathbf{U}\times\delta\mathbf{B}\|\right).\tag{9}$$

Norms of products of components of vector fields worth in round brackets are estimated by means of Hölder's and multiplicative inequalities:

$$\|w_1U_{l2}\kappa\|\ \le\ \|w_1\|_q\|U_{l2}\|_p\ \le 4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}\circ\\ \circ\,\|w_2\beta_2\|^{1/2-1/q}\,\|\nabla\times\delta\mathbf{U}\|^{1/2+1/q}\,\|U_{l2}\beta_2\|^{2/p}\,\|\nabla\times\mathbf{U}_l\|^{1-2/p}=\\ =4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}\|w_2\beta_2\|^{1/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-1/p}\,\|U_{l2}\beta_2\|^{2/p}\,\|\nabla\times\mathbf{U}_l\|^{1-2/p};\\ \|w_2U_{l1}\|\ \le\ \|w_2\|_p\|U_{l1}\|_q\ \le\ 4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}\circ\\ \circ\,\|w_2\beta_2\|^{2/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-2/p}\,\|U_{l2}\beta_2\|^{1/2-1/q}\,\|\nabla\times\mathbf{U}_l\|^{1/2+1/q}=\\ =4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}\|w_2\beta_2\|^{2/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-2/p}\,\|U_{l2}\beta_2\|^{1/2p}\,\|\nabla\times\mathbf{U}_l\|^{1-1/2p};\\ \|\mathbf{U}\times\delta\mathbf{B}\|\ \le\ \|\mathbf{U}\|_p\|\delta\mathbf{B}\|_q\ \le M_q\,\|\mathbf{U}\|_p\|\nabla\times\delta\mathbf{B}\|;\\ \|\delta\mathbf{U}\times\delta\mathbf{B}\|\ \le\ \|\delta\mathbf{U}\|_p\|\delta\mathbf{B}\|_q\ \le\ M_q\|\delta\mathbf{U}\|_p\|\nabla\times\delta\mathbf{B}\|;\\ 2<p\le 3,\quad 1/p+1/q=1/2,\quad 6\le q<\infty,\quad \beta=\max\left(4,\tfrac{p}{p-2}\right).$$

Here

$$\|\delta\mathbf{U}\|_p\le 4\mu d^{3/2p-1/4}\,\|w_2\beta_2\|^{1/4+1/2p}\,\|\nabla\times\delta\mathbf{U}\|^{3/4-1/2p}+\\ +\left(\tfrac{p}{2}\right)^{1-2/p}\|w_2\beta_2\|^{2/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-2/p}\le\\ \le\mu d^{3/2p-1/4}\left(\tfrac{p}{p+2}\varepsilon_1^{-1-2/p}\,\|w_2\beta_2\|+\tfrac{p}{3p-2}\varepsilon_1^{3-2/p}\,\|\nabla\times\delta\mathbf{U}\|\right)+\\ +\left(\tfrac{p}{2}\right)^{1-2/p}\left(\tfrac{2}{p}\varepsilon_2^{-p/2}\,\|w_2\beta_2\|+\tfrac{p-2}{p}\varepsilon_2^{p/(p-2)}\,\|\nabla\times\delta\mathbf{U}\|\right)=\\ =\left[\mu d^{3/2p-1/4}\tfrac{p}{p+2}\varepsilon_1^{-1-2/p}+\left(\tfrac{2}{p}\right)^{2/p}\varepsilon_2^{-p/2}\right]\|w_2\beta_2\|+\\ +\left[\mu d^{3/2p-1/4}\tfrac{p}{3p-2}\varepsilon_1^{3-2/p}+\tfrac{p-2}{p}\left(\tfrac{p}{2}\right)^{1-2/p}\varepsilon_2^{p/(p-2)}\right]\|\nabla\times\delta\mathbf{U}\|.$$

Established estimates and estimates (8) are substituted into (9):

$$\|\nabla\times\delta\mathbf{U}\|^2+H^2\underline{m}^2\|w_2\kappa\|^2\ \le\ 4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}R\|\nabla\times\delta\mathbf{U}\|\circ\\ \circ\left(C_{l\perp}^{2/p}C_{l\nabla}^{1-2/p}H^{1-2/p}\,\|w_2\beta_2\|^{1/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-1/p}+\right.\\ \left.+C_{l\perp}^{1/2p}C_{l\nabla}^{1-1/2p}H^{1/2-1/2p}\,\|w_2\beta_2\|^{2/p}\,\|\nabla\times\delta\mathbf{U}\|^{1-2/p}\right)+\\ +H^{3/2}R_m\left\{\overline{m}M_q\,C_p\,\|w_2\beta_2\|\,H^{1/4-1/2p}+\right.\\ +C_\perp M_q\mu H^{-1/2}\left\{\left[d^{3/2p-1/4}\tfrac{p}{p+2}\varepsilon_1^{-1-2/p}+\left(\tfrac{2}{p}\right)^{2/p}\varepsilon_2^{-p/2}\right]\|w_2\beta_2\|+\right.\\ \left.\left.+\left[d^{3/2p-1/4}\tfrac{p}{3p-2}\varepsilon_1^{3-2/p}+\tfrac{p-2}{p}\left(\tfrac{p}{2}\right)^{1-2/p}\varepsilon_2^{p/(p-2)}\right]\|\nabla\times\delta\mathbf{U}\|\right\}\right\}.\tag{10}$$

From the inequality (10) the required estimates are removed. Availability of estimate for $\|\nabla \times \delta\mathbf{B}\|$ permits to bypass only by "direct" estimates. For their establishing in expressions in the right part of the inequality (10) by means of Jung's inequality estimated norms are allocated. This summands are estimated so:

$$\begin{gathered} \|\nabla\times\delta\mathbf{U}\|^{2-1/p}\|w_2\beta_2\|^{1/p} \leq \\ \leq \tfrac{2p-1}{2p}\varepsilon_3^{2p/(2p-1)}\|\nabla\times\delta\mathbf{U}\|^2+\tfrac{1}{2p}\varepsilon_3^{-2p}\|w_2\beta_2\|^2; \\ \|\nabla\times\delta\mathbf{U}\|^{2-2/p}\|w_2\beta_2\|^{2/p} \leq \\ \leq \tfrac{p-1}{p}\varepsilon_4^{p/(p-1)}\|\nabla\times\delta\mathbf{U}\|^2+\tfrac{1}{p}\varepsilon_4^{-p}\|w_2\beta_2\|^2. \end{gathered}$$

This estimates are substituted into (10). After grouping of summands of the same type the main energetic inequality is established:

$$\begin{gathered} \Big\{1-4\beta\mu\left(\tfrac{p}{2}\right)^{1-2/p}R\Big[\left(1-\tfrac{1}{2p}\right)C_{l\perp}^{2/p}C_{l\nabla}^{1-2/p}\varepsilon_3^{2p/(2p-1)}H^{1-2/p}+ \\ \left(1-\tfrac{1}{p}\right)C_{l\perp}^{1/2p}C_{l\nabla}^{1-1/2p}\varepsilon_4^{p/(p-1)}H^{1/2-1/2p}\Big]\Big\}\|\nabla\times\delta\mathbf{U}\|^2+ \\ +H^2\Big\{\underline{m}^2-4\mu\left(\tfrac{p}{2}\right)^{1-2/p}R\Big[\tfrac{1}{2p}C_{l\perp}^{2/p}C_{l\nabla}^{1-2/p}\varepsilon_3^{-2p}H^{-1-2/p}+ \\ +\tfrac{1}{p}C_{l\perp}^{1/2p}C_{l\nabla}^{1-1/2p}\varepsilon_4^{-p}H^{-3/2-1/2p}\Big]\Big\}\|w_2\kappa\|^2 \leq \\ \leq H^{3/2}R_m\Big\{\overline{m}M_q\,C_p\|w_2\beta_2\|H^{1/4-1/2p}+ \\ +C_\perp M_q\mu H^{-1/2}\Big\{\Big[d^{3/2p-1/4}\tfrac{p}{p+2}\varepsilon_1^{-1-2/p}+\left(\tfrac{2}{p}\right)^{2/p}\varepsilon_2^{-p/2}\Big]\|w_2\beta_2\|+ \\ +\Big[d^{3/2p-1/4}\tfrac{p}{3p-2}\varepsilon_1^{3-2/p}+\tfrac{p-2}{p}\left(\tfrac{p}{2}\right)^{1-2/p}\varepsilon_2^{p/(p-2)}\Big]\|\nabla\times\delta\mathbf{U}\|\Big\}\Big\}. \end{gathered} \tag{11}$$

Numbers $\varepsilon_1, \varepsilon_2$ in this inequality are chosen from the condition of the positiveness of the multiplier at $\|\nabla \times \delta\mathbf{U}\|^2$. Let

$$\begin{gathered} \varepsilon_3=\left[\frac{p}{2p-1}\cdot\frac{H^{-1+2/p}}{8\beta\mu R\left(\frac{p}{2}\right)^{1-2/p}C_{l\perp}^{2/p}C_{l\nabla}^{1-2/p}}\right]^{\frac{2p-1}{2p}}; \\ \varepsilon_4=\left[\frac{p}{p-1}\cdot\frac{H^{-1/2+1/2p}}{8\beta\mu R\left(\frac{p}{2}\right)^{1-2/p}C_{l\perp}^{1/2p}C_{l\nabla}^{1-1/2p}}\right]^{\frac{p-1}{p}}. \end{gathered}$$

Sufficient conditions of the admissibility of $\mathbf{B}_0$-reduction are obtained from the conditions of positiveness of multiplier at $\|w_2\|$ in (11). Let $H \geq \max(H_{01}, H_{02})$, where H_{02} is determined from next condition:

$$\begin{gathered} 2^{4p+1-2/p}p^{-4+2/p}(2p-1)^{2p-1}(\beta\mu)^{2p-1}C_{l\perp}^4C_{l\nabla}^{2p-4}R^{2p-1}H_{02}^{2p-6}+ \\ +2^{2p-2/p}p^{-2+\frac{2}{p}}(p-1)^{p-1}(\beta\mu)^{p-1}C_{l\perp}^{1/2}C_{l\nabla}^{p-1/2}R^{p-1}H_{02}^{-(5-p)/2}\leq 0.75\underline{m}^2. \end{gathered} \tag{12}$$

Then from (11) follows inequality

$$\begin{gathered} 0.25\|\nabla\times\delta\mathbf{U}\|^2+0.25\underline{m}^2H^2\|w_2\kappa\|^2 \leq \\ \leq H^{3/2}R_m\Big\{\overline{m}M_q\,C_p\|w_2\beta_2\|H^{1/4-1/2p}+ \\ +C_\perp M_q\mu H^{-1/2}\Big\{\Big[d^{3/2p-1/4}\tfrac{p}{p+2}\varepsilon_1^{-1-2/p}+\left(\tfrac{2}{p}\right)^{2/p}\varepsilon_2^{-p/2}\Big]\|w_2\beta_2\|+ \\ +\Big[d^{3/2p-1/4}\tfrac{p}{3p-2}\varepsilon_1^{3-2/p}+\tfrac{p-2}{p}\left(\tfrac{p}{2}\right)^{1-2/p}\varepsilon_2^{p/(p-2)}\Big]\|\nabla\times\delta\mathbf{U}\|\Big\}\Big\}. \end{gathered} \tag{13}$$

In this inequality it should to determine values of numbers $\varepsilon_2, \varepsilon_2$. Let

$$\varepsilon_1 = H^{-\frac{p}{2(3p-2)}}, \quad \varepsilon_2^{p/(p-2)} = H^{-\frac{p-2}{2p}}.$$

After allocation of the complete squares from the inequality (13) next estimates are established:

$$\begin{gathered}
\|\nabla \times \delta \mathbf{U}\| \leq 2C_\perp M_q \mu R_m H^{1/2} \left[d^{3/2p-1/4} \tfrac{p}{3p-2} + (p-2) \left(\tfrac{2}{p}\right)^{2/p} \right] + \\
+ R_m H^{1/2} \underline{m}^{-1} \left\{ \overline{m} M_q \; C_p H^{3/4-1/2p} + \right. \\
\left. + C_\perp M_q \mu \left[d^{3/2p-1/4} \tfrac{p}{p+2} H^{\frac{p+2}{2(3p-2)}} + \left(\tfrac{2}{p}\right)^{2/p} H^{\frac{p-2}{4}} \right] \right\}, \\
\|w_2 \kappa\| \; \leq C_\perp M_q \mu R_m H^{-1} \left[d^{3/2p-1/4} \tfrac{p}{3p-2} + \tfrac{p-2}{p} \left(\tfrac{p}{2}\right)^{1-2/p} \right] + \\
+ 2R_m \underline{m}^{-1} \left\{ \overline{m} M_q \; C_p H^{-1/4-1/2p} + \right. \\
\left. + C_\perp M_q \mu H^{-1} \left[d^{3/2p-1/4} \tfrac{p}{p+2} H^{\frac{p+2}{2(3p-2)}} + \left(\tfrac{2}{p}\right)^{2/p} H^{\frac{p-2}{4}} \right] \right\}.
\end{gathered} \tag{14}$$

In this estimates number p, obviously, should choose reasonably close two. From the estimates (14) and first inequality (6) follows next estimate for residual of the vector field $\delta \mathbf{U}$:

$$\begin{gathered}
\|\delta \mathbf{U}\| \leq (2.5d)^{1/2} R_m \left\{ 2C_\perp M_q \mu \left[d^{3/2p-1/4} \tfrac{p}{3p-2} + \tfrac{p-2}{p} \left(\tfrac{p}{2}\right)^{1-2/p} \right] + \right. \\
+ \underline{m}^{-1} \left\{ \overline{m} M_q \; C_p H^{3/4-1/2p} + \right. \\
\left. + C_\perp M_q \mu H^{-1/2} \left[d^{3/2p-1/4} \tfrac{p}{p+2} H^{\frac{p+2}{2(3p-2)}} + \left(\tfrac{2}{p}\right)^{2/p} H^{\frac{p-2}{4}} \right] \right\}.
\end{gathered} \tag{15}$$

For the sufficiently large significances of H obtained estimates can be copied so:

$$\begin{gathered}
\|\nabla \times \delta \mathbf{U}\| \leq \; R_m C_{\delta\nabla}(R, D, S) H^{1/2+\gamma}; \\
\|w_2 \kappa\| \; \leq R_m C_{\delta\perp}(R, D, S) H^{-1/2+\gamma}; \\
\|\delta \mathbf{U}\| \; \leq R_m C_\delta(R, D, S) H^{\gamma},
\end{gathered} \tag{16}$$

where γ there is no matter how small positive number.

From (12) follows that conditions of allowability of $\mathbf{B}_0-$reduction depend not only from numbers R_m and H, but from from the Reynolds number R too. Conditions (12) give following estimate of the significances of R and H_0, at which ininductional approximation is allowable: $H_0 = C_h R^{2+\delta}$ $\delta > 0$. From estimates (15) follows that there are MHD-flows, for which A.B. Tsinober's hypothesis, as appear, is incorrect.

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