

Lateral ideals of ternary semigroups

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Abstract. In this paper, we give some auxiliary results are also necessary for our considerations and characterize the relationship between the (0-)minimal and maximal lateral ideals and the lateral simple and lateral 0-simple ternary semigroups analogous to the characterizations of minimal and maximal left ideals in ordered semigroups considered by Cao and Xu [5].

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1. Preliminaries

In 1995, Dixit and Dewan [4] introduced and studied the properties of (quasi-, bi-, left, right) lateral ideals in ternary semigroups. In 2000, Cao and Xu [5] characterized the minimal and maximal left ideals in ordered semigroups and gave some characterizations of minimal and maximal left ideals in ordered semigroups. In 2002, Arslanov and Kehayopulu [3] characterized the minimal and maximal ideals in ordered semigroups.

The concept of the minimality and maximality of (left) ideals is the really interested and important thing about (ordered) semigroups. Now we also characterize the minimality and maximality of lateral ideals in ternary semigroups and give some characterizations of the minimality and maximality of lateral ideals in ternary semigroups analogous to the characterizations of the minimality and maximality of (left) ideals in (ordered) semigroups.

Our purpose in this paper is fourfold.

- (1) To introduce the concept of lateral simple and lateral 0-simple ternary semigroups.

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- (2) To characterize the properties of lateral ideals in ternary semigroups.
- (3) To characterize the relationship between the (0-)minimal lateral ideals and the left (0-)simple ternary semigroups.
- (4) To characterize the relationship between the maximal lateral ideals and the lateral simple and lateral 0-simple ternary semigroups.

To present the main theorems we first recall the definition of a ternary semigroup which is important here.

A nonempty set T is called a *ternary semigroup* [1] if there exists a ternary operation $T \times T \times T \rightarrow T$, written as $(x_1, x_2, x_3) \rightarrow [x_1x_2x_3]$, satisfying the following identity for any $x_1, x_2, x_3, x_4, x_5 \in T$,

$$[[x_1x_2x_3]x_4x_5] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]].$$

Example 1.1 ([4]). Let $T = \{-i, 0, i\}$. Then T is a ternary semigroup under the multiplication over complex number while T is not a semigroup under complex number multiplication.

Example 1.2 ([4]). Let

$$T = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

Then T is a ternary semigroup under matrix multiplication.

For nonempty subsets A, B and C of T , let

$$[ABC] := \{[abc] : a \in A, b \in B \text{ and } c \in C\}.$$

If $A = \{a\}$, then we also write $[\{a\}BC]$ as $[aBC]$, and similarly if $B = \{b\}$ or $C = \{c\}$ or $A = \{a\}$ and $B = \{b\}$ or $A = \{a\}$ and $C = \{c\}$ or $B = \{b\}$ and $C = \{c\}$. A nonempty subset S of a ternary semigroup T is called a *ternary subsemigroup* [4] of T if $[SSS] \subseteq S$. A nonempty subset M of a ternary semigroup T is called a *lateral ideal* [4] of T if $[TMT] \subseteq M$. A lateral ideal M of a ternary semigroup S is called a *proper lateral ideal* of T if $M \neq T$. The intersection of all lateral ideals of a ternary subsemigroup S of a ternary semigroup T containing a nonempty subset A of S is the *lateral ideal of S generated by A* . For $A = \{a\}$, let $M_S(a)$ denote the lateral ideal of S generated by $\{a\}$. If

$S = T$, then we also write $M_T(a)$ as $M(a)$. An element a of a ternary semigroup T with at least two elements is called a *zero element* of T if $[at_1t_2] = [t_1at_2] = [t_1t_2a] = a$ for all $t_1, t_2 \in T$ and denote it by 0 . If T is a ternary semigroup with zero, then every lateral ideal of T contains a zero element. A ternary semigroup T without zero is called *lateral simple* if it has no proper lateral ideals. A ternary semigroup T with zero is called *lateral 0-simple* if it has no nonzero proper lateral ideals and $[TTT] \neq \{0\}$. A lateral ideal M of a ternary semigroup T without zero is called a *minimal lateral ideal* of T if there is no a lateral ideal A of T such that $A \subset M$. Equivalently, if for any lateral ideal A of T such that $A \subseteq M$, we have $A = M$. A nonzero lateral ideal M of a ternary semigroup T with zero is called a *0-minimal lateral ideal* of T if there is no a nonzero lateral ideal A of T such that $A \subset M$. Equivalently, if for any nonzero lateral ideal A of T such that $A \subseteq M$, we have $A = M$. Equivalently, if for any lateral ideal A of T such that $A \subset M$, we have $A = \{0\}$. A proper lateral ideal M of a ternary semigroup T is called a *maximal lateral ideal* of T if for any lateral ideal A of T such that $M \subset A$, we have $A = T$. Equivalently, if for any proper lateral ideal A of T such that $M \subseteq A$, we have $A = M$.

For any positive integers m and n with $m \leq n$ and any elements x_1, x_2, \dots, x_{2n} and x_{2n+1} of a ternary semigroup T [2], we can write

$$\begin{aligned} [x_1x_2 \dots x_{2n+1}] &= [x_1 \dots x_mx_{m+1}x_{m+2} \dots x_{2n+1}] \\ &= [x_1 \dots [x_mx_{m+1}x_{m+2}]x_{m+3}x_{m+4} \dots x_{2n+1}]. \end{aligned}$$

Throughout this paper, T stands for a ternary semigroup. The following two lemmas are also necessary for our considerations and easy to verify.

Lemma 1.1 ([4]). For any nonempty subset A of T , $[TTATT] \cup [TAT] \cup A$ is the smallest lateral ideal of T containing A .

Furthermore, for any $a \in T$,

$$M(a) = [TTaTT] \cup [TaT] \cup \{a\}.$$

Lemma 1.2. For any nonempty subset A of T , $[TTATT] \cup [TAT]$ is a lateral ideal of T .

Lemma 1.3. If T has no a zero element, then the following statements are equivalent.

- (a) T is lateral simple.
- (b) $[TTaTT] \cup [TaT] = T$ for all $a \in T$.

(c) $M(a) = T$ for all $a \in T$.

Proof. By Lemma 1.2 and T is lateral simple, we have $[TTaTT] \cup [TaT] = T$ for all $a \in T$. Therefore (a) implies (b). By Lemma 1.1, we have $M(a) = [TTaTT] \cup [TaT] \cup \{a\} = T \cup \{a\} = T$. Thus (b) implies (c). Now let M be a lateral ideal of T and let $a \in M$. Then $T = M(a) \subseteq M \subseteq T$, so $M = T$. Hence T is lateral simple, we have that (c) implies (a).

Hence the proof is completed. \square

Lemma 1.4. If T has a zero element, then the following statements hold.

(a) If T is lateral 0-simple, then $M(a) = T$ for all $a \in T \setminus \{0\}$.

(b) If $M(a) = T$ for all $a \in T \setminus \{0\}$, then either $[TTT] = \{0\}$ or T is lateral 0-simple.

Proof. (a) Assume that T is lateral 0-simple. Then $M(a)$ is a nonzero lateral ideal of T for all $a \in T \setminus \{0\}$. Hence $M(a) = T$ for all $a \in T \setminus \{0\}$.

(b) Assume that $M(a) = T$ for all $a \in T \setminus \{0\}$ and let $[TTT] \neq \{0\}$. Now let M be a nonzero lateral ideal of T and put $a \in M \setminus \{0\}$. Then $T = M(a) \subseteq M \subseteq T$, so $M = T$. Therefore T is lateral 0-simple.

Therefore we complete the proof of the lemma. \square

The next lemma is easy to verify.

Lemma 1.5. Let $\{M_\gamma : \gamma \in \Gamma\}$ be a family of lateral ideals of T . Then $\bigcup_{\gamma \in \Gamma} M_\gamma$ is a lateral ideal of T and $\bigcap_{\gamma \in \Gamma} M_\gamma$ is also a lateral ideal of T if $\bigcap_{\gamma \in \Gamma} M_\gamma \neq \emptyset$.

Lemma 1.6. If M is a lateral ideal of T and S is a ternary subsemigroup of T , then the following statements hold.

(a) If S is lateral simple such that $S \cap M \neq \emptyset$, then $S \subseteq M$.

(b) If S is lateral 0-simple such that $S \setminus \{0\} \cap M \neq \emptyset$, then $S \subseteq M$.

Proof. (a) Assume that S is lateral simple such that $S \cap M \neq \emptyset$. Then, let $a \in S \cap M$. By Lemma 1.2, we have $([SSaSS] \cup [SaS]) \cap S$ is a lateral ideal of S . This implies that $([SSaSS] \cup [SaS]) \cap S = S$. Hence $S \subseteq [SSaSS] \cup [SaS] \subseteq [TTMTT] \cup [TMT] \subseteq [TMT] \subseteq M$, so $S \subseteq M$.

(b) Assume that S is lateral 0-simple such that $S \setminus \{0\} \cap M \neq \emptyset$. Then, let $a \in S \setminus \{0\} \cap M$. By Lemmas 1.1 and 1.4 (a), we have $S = M_S(a) = ([SSaSS] \cup [SaS] \cup \{a\}) \cap S \subseteq [SSaSS] \cup [SaS] \cup \{a\} \subseteq [TTaTT] \cup [TaT] \cup \{a\} = M(a) \subseteq M$. Therefore $S \subseteq M$.

Hence the proof of the lemma is completed. \square

Lemma 1.7. If A is a nonempty subset of a lateral ideal M of T such that $[MMAMM] = [MAM]$, then $[MAM]$ is a lateral ideal of T .

Proof. Assume that A is a nonempty subset of a lateral ideal M of T such that $[MMAMM] = [MAM]$. Then $[TMT] \subseteq M$. Hence $[T[MAM]T] = [T[MMAMM]T] = [[TMM]A[MMT]] \subseteq [[TMT]A[TMT]] \subseteq [MAM]$. Therefore $[MAM]$ is a lateral ideal of T . \square

2. Minimality of lateral ideals

In this section, we characterize the relationship between the (0-)minimal lateral ideals and the lateral simple and lateral 0-simple ternary semi-groups.

Theorem 2.1. If T has no a zero element and M is a lateral ideal of T , then the following statements hold.

- (a) If M is a minimal lateral ideal without zero of T , then either there exists a lateral ideal A of M such that $[MMAMM] \neq [MAM]$ or M is lateral simple.
- (b) If M is lateral simple, then M is a minimal lateral ideal of T .
- (c) If M is a minimal lateral ideal with zero of T , then either there exists a nonzero lateral ideal A of M such that $[MMAMM] \neq [MAM]$ or M is lateral 0-simple.

Proof. (a) Assume that M is a minimal lateral ideal without zero of T and $[MMAMM] = [MAM]$ for all lateral ideals A of M . Now let A be a lateral ideal of M . Then $[MMAMM] = [MAM] \subseteq A \subseteq M$. By Lemma 1.7, we have $[MAM]$ is a lateral ideal of T . Since M is a minimal lateral ideal of T , $[MAM] = M$. Therefore $A = M$, so we conclude that M is lateral simple.

(b) Assume that M is lateral simple. Let A be a lateral ideal of T such that $A \subseteq M$. Then $A \cap M \neq \emptyset$, it follows from Lemma 1.6 (a) that $M \subseteq A$. Hence $A = M$, so M is a minimal lateral ideal of T .

(c) It is similar to the proof of statement (a).

Therefore we complete the proof of the theorem. \square

Using the similar proof of Theorem 2.1 (a) and the Lemma 1.6 (b), we have Theorem 2.2.

Theorem 2.2. If T has a zero element and M is a nonzero lateral ideal of T , then the following statements hold.

- (a) If M is a 0-minimal lateral ideal of T , then either there exists a nonzero lateral ideal A of M such that $[MMAMM] \neq [MAM] = \{0\}$ or M is lateral 0-simple.
- (b) If M is lateral 0-simple, then M is a 0-minimal lateral ideal of T .

Theorem 2.3. If T has no a zero element but it has proper lateral ideals, then every proper lateral ideal of T is minimal if and only if T contains exactly one proper lateral ideal or T contains exactly two proper lateral ideals M_1 and M_2 , $M_1 \cup M_2 = T$ and $M_1 \cap M_2 = \emptyset$.

Proof. Assume that every proper lateral ideal of T is minimal. Now let M be a proper lateral ideal of T . Then M is a minimal lateral ideal of T . We consider the following two cases:

Case 1: $T = M(a)$ for all $a \in T \setminus M$.

If K is also a proper lateral ideal of T and $K \neq M$, then $K \setminus M \neq \emptyset$ because M is a minimal lateral ideal of T . Thus there exists $a \in K \setminus M \subseteq T \setminus M$. Hence $T = M(a) \subseteq K \subseteq T$, so $K = T$. It is impossible, so $K = M$. In this case, M is the unique proper lateral ideal of T .

Case 2: There exists $a \in T \setminus M$ such that $T \neq M(a)$.

Then $M(a) \neq M$ and $M(a)$ is a minimal lateral ideal of T . By Lemma 1.5, $M(a) \cup M$ is a lateral ideal of T . By hypothesis and $M \subset M(a) \cup M$, we get $M(a) \cup M = T$. Since $M(a) \cap M \subset M(a)$ and $M(a)$ is a minimal lateral ideal of T , $M(a) \cap M = \emptyset$. Now let K be an arbitrary proper lateral ideal of T . Then K is a minimal lateral ideal of T . We observe that $K = K \cap T = K \cap (M(a) \cup M) = (K \cap M(a)) \cup (K \cap M)$. If $K \cap M \neq \emptyset$, then $K = M$ because K and M are minimal lateral ideals of T . If $K \cap M(a) \neq \emptyset$, then $K = M(a)$ because K and $M(a)$ are minimal lateral ideals of T . In this case, T contains exactly two proper lateral ideals M and $M(a)$, $M(a) \cup M = T$ and $M(a) \cap M = \emptyset$.

The converse is obvious. □

Using the same proof of Theorem 2.3, we have Theorem 2.4.

Theorem 2.4. If T has a zero element and nonzero proper lateral ideals, then every nonzero proper lateral ideal of T is 0-minimal if and only if T contains exactly one nonzero proper lateral ideal or T contains exactly two nonzero proper lateral ideals M_1 and M_2 , $M_1 \cup M_2 = T$ and $M_1 \cap M_2 = \{0\}$.

3. Maximality of lateral ideals

In this section, we characterize the relationship between the maximality of lateral ideals and the union \mathcal{U} of all (nonzero) proper lateral ideals in ternary semigroups.

Theorem 3.1. If T has no a zero element but it has proper lateral ideals, then every proper lateral ideal of T is maximal if and only if T contains exactly one proper lateral ideal or T contains exactly two proper lateral ideals M_1 and M_2 , $M_1 \cup M_2 = T$ and $M_1 \cap M_2 = \emptyset$.

Proof. Assume that every proper lateral ideal of T is maximal. Now let M be a proper lateral ideal of T . Then M is a maximal lateral ideal of T . We consider the following two cases:

Case 1: $T = M(a)$ for all $a \in T \setminus M$.

If K is also a proper lateral ideal of T and $K \neq M$, then K is a maximal lateral ideal of T . This implies that $K \setminus M \neq \emptyset$, so there exists $a \in K \setminus M \subseteq T \setminus M$. Thus $T = M(a) \subseteq K \subseteq T$, so $K = T$. It is impossible, so $K = M$. In this case, M is the unique proper lateral ideal of T .

Case 2: There exists $a \in T \setminus M$ such that $T \neq M(a)$.

Then $M(a) \neq M$ and $M(a)$ is a maximal lateral ideal of T . By Lemma 1.5, $M(a) \cup M$ is a lateral ideal of T . Since $M \subset M(a) \cup M$ and M is a maximal lateral ideal of T , $M(a) \cup M = T$. By hypothesis and $M(a) \cap M \subset M(a)$, we get $M(a) \cap M = \emptyset$. Now let K be an arbitrary proper lateral ideal of T . Then K is a maximal lateral ideal of T . We observe that $K = K \cap T = K \cap (M(a) \cup M) = (K \cap M(a)) \cup (K \cap M)$. If $K \cap M \neq \emptyset$, then $K = M$ because $K \cap M$ and M are maximal lateral ideals of T . If $K \cap M(a) \neq \emptyset$, then $K = M(a)$ because $K \cap M(a)$ and $M(a)$ are maximal lateral ideals of T . In this case, T contains exactly two proper lateral ideals M and $M(a)$, $M(a) \cup M = T$ and $M(a) \cap M = \emptyset$.

The converse is obvious. \square

Using the same proof of Theorem 3.1, we have Theorem 3.2.

Theorem 3.2. If T has a zero element and nonzero proper lateral ideals, then every nonzero proper lateral ideal of T is maximal if and only if T contains exactly one nonzero proper lateral ideal or T contains exactly two nonzero proper lateral ideals M_1 and M_2 , $M_1 \cup M_2 = T$ and $M_1 \cap M_2 = \{0\}$.

Theorem 3.3. A proper lateral ideal M of T is maximal if and only if

(a) $T \setminus M = \{a\}$ and $[TaT] \subseteq M$ for some $a \in T$ or

(b) $T \setminus M \subseteq [TTaTT] \cup [TaT]$ for all $a \in T \setminus M$.

Proof. Assume that M is a maximal lateral ideal of T . Then we consider the following two cases:

Case 1: There exists $a \in T \setminus M$ such that $[TTaTT] \cup [TaT] \subseteq M$.

Then $[TaT] \subseteq M$. By Lemma 1.1, we have $M \cup \{a\} = (M \cup [TTaTT] \cup [TaT]) \cup \{a\} = M \cup ([TTaTT] \cup [TaT] \cup \{a\}) = M \cup M(a)$. Thus $M \cup \{a\}$ is a lateral ideal of T because $M \cup M(a)$ is a lateral ideal of T . Since M is a maximal lateral ideal of T and $M \subset M \cup \{a\}$, we have $M \cup \{a\} = T$. Hence $T \setminus M = \{a\}$. In this case, the condition (a) is satisfied.

Case 2: $[TTaTT] \cup [TaT] \not\subseteq M$ for all $a \in T \setminus M$.

If $a \in T \setminus M$, then $[TTaTT] \cup [TaT] \not\subseteq M$ and $[TTaTT] \cup [TaT]$ is a lateral ideal of T by Lemma 1.2. By Lemma 1.5, we have $M \cup [TTaTT] \cup [TaT]$ is a lateral ideal of T and $M \subset M \cup [TTaTT] \cup [TaT]$. Since M is a maximal lateral ideal of T , $M \cup [TTaTT] \cup [TaT] = T$. Hence we conclude that $T \setminus M \subseteq [TTaTT] \cup [TaT]$ for all $a \in T \setminus M$. In this case, the condition (b) is satisfied.

Conversely, let J be a lateral ideal of T such that $M \subset J$. Then $J \setminus M \neq \emptyset$. If $T \setminus M = \{a\}$ and $[TaT] \subseteq M$ for some $a \in T$, then $J \setminus M \subseteq T \setminus M = \{a\}$. Thus $J \setminus M = \{a\}$, so $J = M \cup \{a\} = T$. Hence M is a maximal lateral ideal of T . If $T \setminus M \subseteq [TTaTT] \cup [TaT]$ for all $a \in T \setminus M$, then $T \setminus M \subseteq [TTxTT] \cup [TxT] \subseteq [TTJTT] \cup [TJT] \subseteq J$ for all $x \in J \setminus M$. Hence $T = (T \setminus M) \cup M \subseteq J \subseteq T$, so $J = T$. Therefore M is a maximal lateral ideal of T .

Hence the theorem is now completed. \square

For a ternary semigroup T , let \mathcal{U} denote the union of all nonzero proper lateral ideals of T if T has a zero element and let \mathcal{U} denote the union of all proper lateral ideals of T if T has no a zero element. Then it is easy to verify Lemma 3.1.

Lemma 3.1. $\mathcal{U} = T$ if and only if $M(a) \neq T$ for all $a \in T$.

As a consequence of Theorem 3.3 and Lemma 3.1, we obtain the next two theorems.

Theorem 3.4. If T has no a zero element, then one and only one of the following four conditions is satisfied.

(a) T is lateral simple.

- (b) $M(a) \neq T$ for all $a \in T$.
- (c) There exists $a \in T$ such that $M(a) = T$, $a \notin [TTaTT] \cup [TaT]$, $[TaT] \subseteq \mathcal{U} = T \setminus \{a\}$ and \mathcal{U} is the unique maximal lateral ideal of T .
- (d) $T \setminus \mathcal{U} = \{x \in T : [TTxTT] \cup [TxT] = T\}$ and \mathcal{U} is the unique maximal lateral ideal of T .

Proof. Assume that T is not lateral simple. Then there exists a proper lateral ideal of T , so \mathcal{U} is a lateral ideal of T . We consider the following two cases:

Case 1: $\mathcal{U} = T$.

By Lemma 3.1, we have $M(a) \neq T$ for all $a \in T$. In this case, the condition (b) is satisfied.

Case 2: $\mathcal{U} \neq T$.

Then \mathcal{U} is a maximal lateral ideal of T . Now assume that M is a maximal lateral ideal of T . Then $M \subseteq \mathcal{U} \subset T$ because M is a proper lateral ideal of T . Since M is a maximal lateral ideal of T , we have $M = \mathcal{U}$. Hence \mathcal{U} is the unique maximal lateral ideal of T . By Theorem 3.3, we get

- (i) $T \setminus \mathcal{U} = \{a\}$ and $[TaT] \subseteq \mathcal{U}$ for some $a \in T$ or
- (ii) $T \setminus \mathcal{U} \subseteq [TTaTT] \cup [TaT]$ for all $a \in T \setminus \mathcal{U}$.

Suppose that $T \setminus \mathcal{U} = \{a\}$ and $[TaT] \subseteq \mathcal{U}$ for some $a \in T$. Then $[TaT] \subseteq \mathcal{U} = T \setminus \{a\}$. Since $a \notin \mathcal{U}$, we have $M(a) = T$. If $a \in [TTaTT] \cup [TaT]$, then $\{a\} \subseteq [TTaTT] \cup [TaT]$. By Lemma 1.1, we have $T = M(a) = [TTaTT] \cup [TaT] \cup \{a\} = [TTaTT] \cup [TaT] \subseteq [T\mathcal{U}T] \cup \mathcal{U} = \mathcal{U} \subseteq T$. Thus $T = \mathcal{U}$, so it is impossible. Hence $a \notin [TTaTT] \cup [TaT]$. In this case, the condition (c) is satisfied.

Now suppose that $T \setminus \mathcal{U} \subseteq [TTaTT] \cup [TaT]$ for all $a \in T \setminus \mathcal{U}$. To show that $T \setminus \mathcal{U} = \{x \in T : [TTxTT] \cup [TxT] = T\}$, let $x \in T \setminus \mathcal{U}$. Then $x \in [TTxTT] \cup [TxT]$, so $\{x\} \subseteq [TTxTT] \cup [TxT]$. By Lemma 1.1, we have $M(x) = [TTxTT] \cup [TxT] \cup \{x\} = [TTxTT] \cup [TxT]$. Since $x \notin \mathcal{U}$, we have $M(x) = T$. Hence $T = M(x) = [TTxTT] \cup [TxT]$. Conversely, let $x \in T$ be such that $[TTxTT] \cup [TxT] = T$. If $x \in \mathcal{U}$, then $M(x) \subseteq \mathcal{U} \subset T$. By Lemma 1.1, we have $M(x) = [TTxTT] \cup [TxT] \cup \{x\} = T \cup \{x\} = T$. It is impossible, so $x \in T \setminus \mathcal{U}$. Hence we conclude that $T \setminus \mathcal{U} = \{x \in T : [TTxTT] \cup [TxT] = T\}$. In this case, the condition (d) is satisfied.

Hence the proof of the theorem is completed. □

Using the same proof of Theorem 3.4, we have Theorem 3.5.

Theorem 3.5. If T has a zero element and $[TTT] \neq \{0\}$, then one and only one of the following four conditions is satisfied.

- (a) T is lateral 0-simple.
- (b) $M(a) \neq T$ for all $a \in T$.
- (c) There exists $a \in T$ such that $M(a) = T$, $a \notin [TTaTT] \cup [TaT]$, $[TaT] \subseteq \mathcal{U} = T \setminus \{a\}$ and \mathcal{U} is the unique maximal lateral ideal of T .
- (d) $T \setminus \mathcal{U} = \{x \in T : [TTxTT] \cup [TxT] = T\}$ and \mathcal{U} is the unique maximal lateral ideal of T .

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